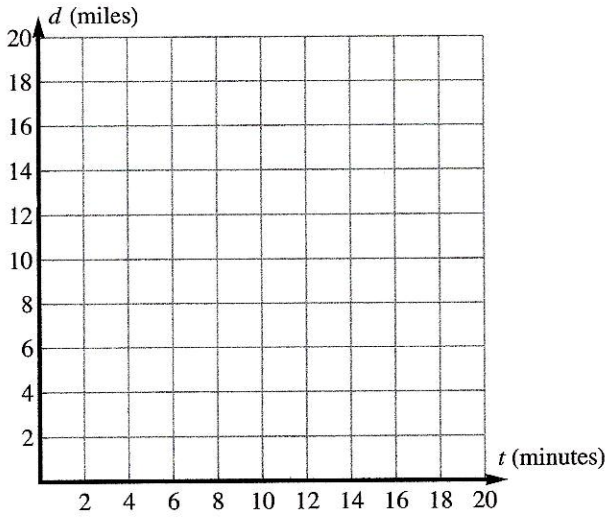
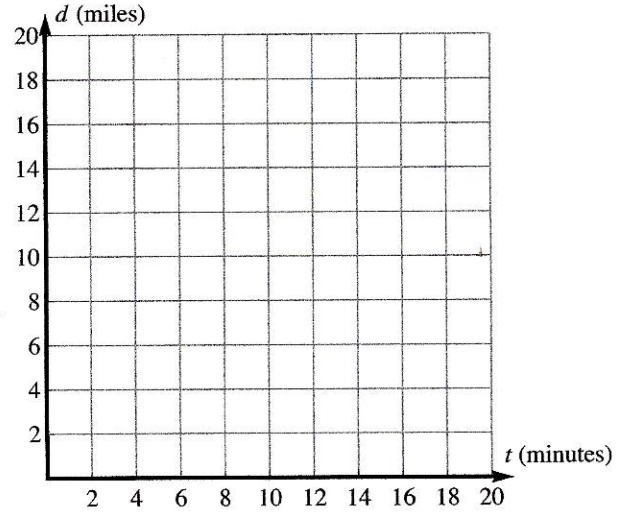
Average Rate of Change vs. Instantaneous Rate of Change

1. Beginning $2\frac{1}{3}$ miles from home, Jonathan drove away from home at a constant rate for 20 minutes. If his constant rate is 35 miles per hour, how far is he from home at the end of the 20 minutes? Draw a graph to model his distance from home during the 20 minute time period.



2. Susan, Jonathan's sister, also drove away from home beginning $2\frac{1}{3}$ miles from home and following the same path as Jonathan. Susan kept varying her velocity by frequently speeding up and slowing down. She arrived at the same location as Jonathan at the end of 20 minutes. To model Susan's distance from home during the 20 minute time period, draw a smooth curve without any sharp corners.



3. Calculate the average velocity for both drivers by calculating the change in position divided by the change in time. These two calculations have the same value; explain why this makes sense. Compare your answers to the rate given in question 1.

4. On the graph showing Susan's position, draw the line segment connecting the point at $t = 0$ and the point at $t = 20$. What is the slope of this line segment and what are the units for the slope? How does this slope compare to the slope of the line that modeled Jonathan's distance from home?

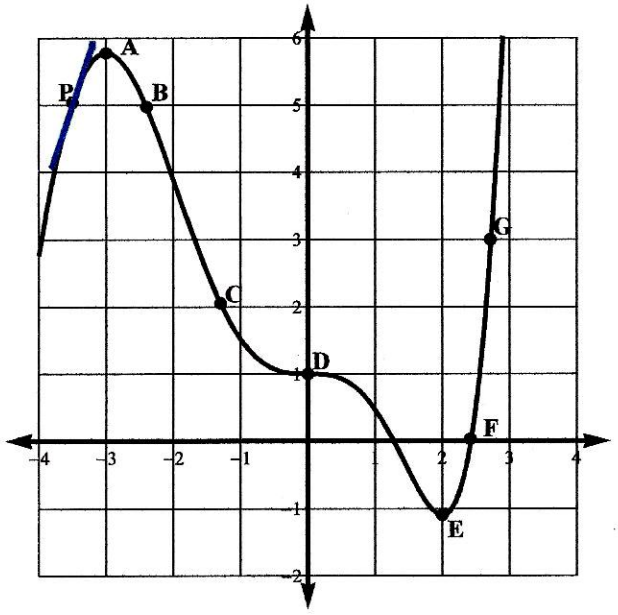
5. If the speed limit over the entire path is 35 miles per hour, did Susan ever drive over the speed limit? Explain your answer by referring to Susan's graph.

6. For non-linear position functions, the exact velocity at a particular time, called instantaneous rate of change or instantaneous velocity, cannot be calculated precisely without the tools of calculus. However, the velocity can be estimated by approximating the slope of a short line segment drawn tangent to the curve at the particular time. On the graph showing Susan's distance, locate at least one time when Susan's instantaneous velocity has the same value as the average velocity.

   (Position a straightedge on the graph so that it is parallel to the line segment drawn on the curve in question 4. Move the straightedge around on the graph keeping the slope of the straightedge fixed. When the straightedge appears to be tangent to the curve, mark the point(s) and sketch a short segment tangent to the curve. At these point(s), the instantaneous velocity is the same as the average velocity.)

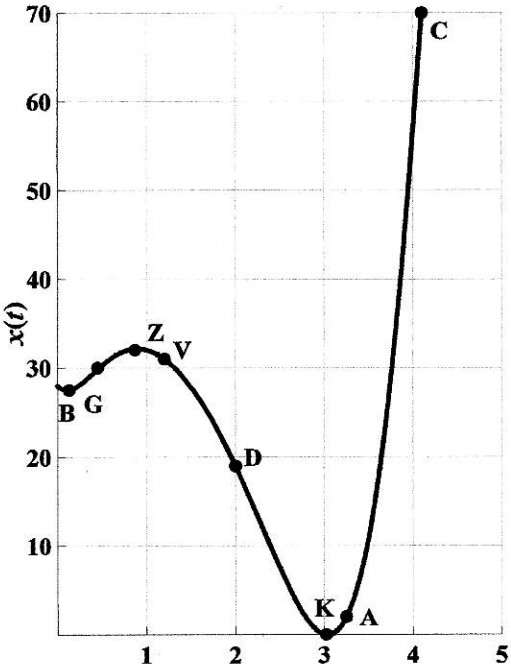
7. Using the function $g(x)$ shown in the graph, draw a small tangent line segment at each labeled point. A small tangent line is drawn at point P as a sample. There is not enough information to draw the segments perfectly, so sketches may vary slightly. Match the slope at each labeled point on the curve with an approximate rate of change value in the table. The slope at each point is called the "instantaneous rate of change" at a point because it is the rate of change at that one instant in time.

Hint: The slope may be the same at different places along the graph.



Rate of Change at Point	Letter
–2	
0	
3	p
6	
15	

8. The graph represents the position $x(t)$ in inches of an object that is moving along a line extending perpendicularly from a wall at a given time, t, measured in seconds. The distance between the object and the wall is indicated on the vertical axis, while time is measured on the horizontal axis.



Point	t	$x(t)$
B	0.13	27.5
G	0.45	30
Z	0.87	32
V	1.2	31
D	2.0	19
K	3.02	0
A	3.25	2
C	4.1	70

a. What do the coordinates of B (0.13, 27.5) and D (2, 19) represent in the context of this situation?

b. Mark small tangent line segments on each of the points that are named. Using these tangent segments, for which point(s) is the instantaneous rate of change negative? What do you know about the motion of the object if the instantaneous rate of change is negative?

c. Observing the tangent segments, over which time intervals is the object moving away from the wall? What do the slopes of these line segments mean in the context of the position function?

d. At which point(s) has the object stopped moving? Describe the slope of the tangent line(s) at the point(s).

e. Speed indicates how fast an object is moving without regard to direction. Order the speeds at the following points from least to greatest: D, G, Z. Explain your reasoning.