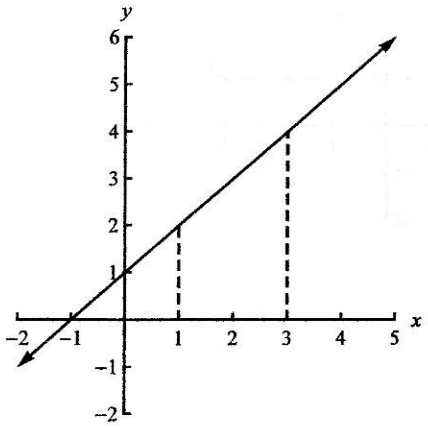
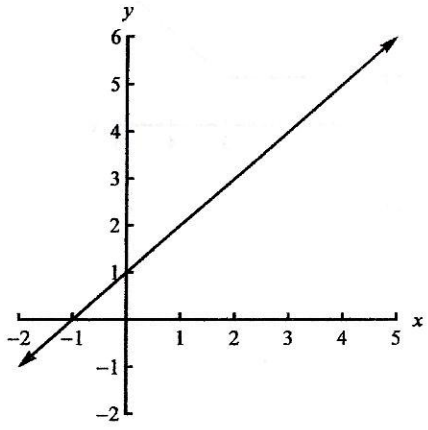
Investigating Area Under a Curve

1. For the given interval, determine the area between the graph of the given function and the x-axis:

a. Interval: $1 \le x \le 3$
Function: $y = x + 1$

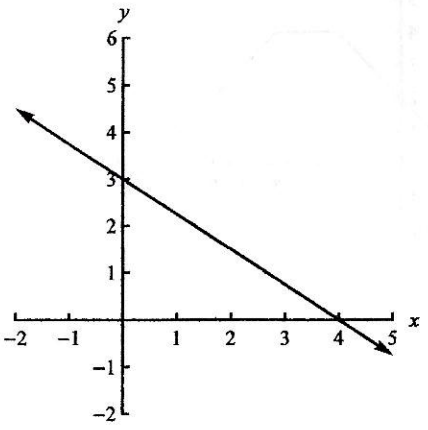


b. Interval: $2 \le x \le 4$
Function: $y = x + 1$



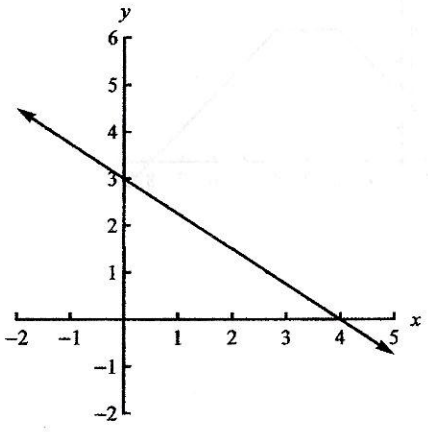
c. Interval: $1 \le x \le 4$

Function: $y = -\frac{3}{4}x + 3$



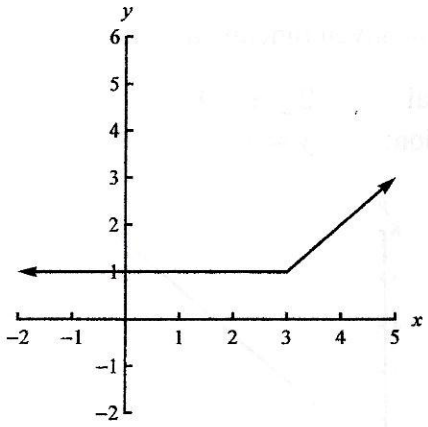
d. Interval: $-1 \le x \le 3$

Function: $y = -\frac{3}{4}x + 3$



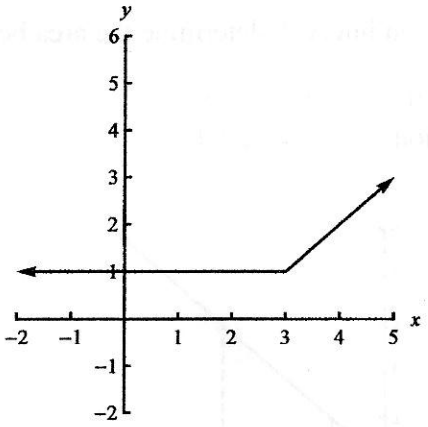
e. Interval: $0 \le x \le 5$

Function: $f(x) = \begin{cases} 1, & x \le 3 \\ x-2, & x \ge 3 \end{cases}$



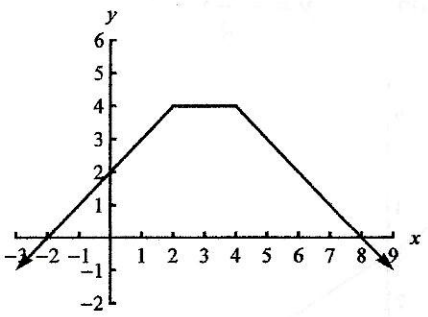
f. Interval: $-2 \le x \le 4$

Function: $f(x) = \begin{cases} 1, & x \le 3 \\ x-2, & x \ge 3 \end{cases}$



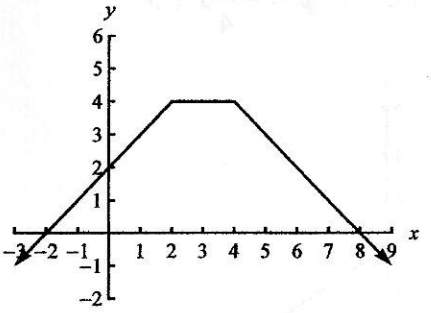
g. Interval: $0 \le x \le 6$

Function: $f(x) = \begin{cases} x+2, & x \le 2 \\ 4, & 2 \le x \le 4 \\ 8-x, & x \ge 4 \end{cases}$



h. Interval: $0 \le x \le 8$

Function: $f(x) = \begin{cases} x+2, & x \le 2 \\ 4, & 2 \le x \le 4 \\ 8-x, & x \ge 4 \end{cases}$



2. Approximate the area between the function and the x-axis for the interval shown on the graph and in the table:

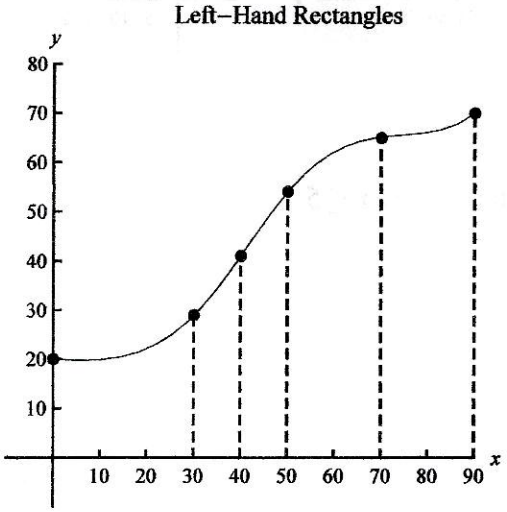
   i. by calculating the sum of left-hand rectangles with widths determined by the data in the table.

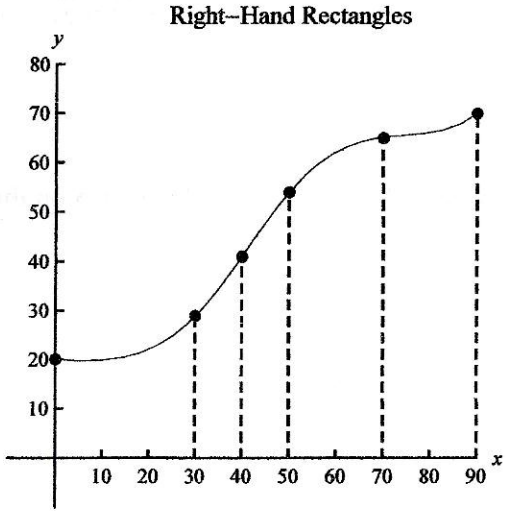
   ii. by calculating the sum of right-hand rectangles with widths determined by the data in the table.

   iii. by averaging the sums of the left-hand and right-hand rectangles and explaining why this may be a better approximation for the actual area under the curve.

a.

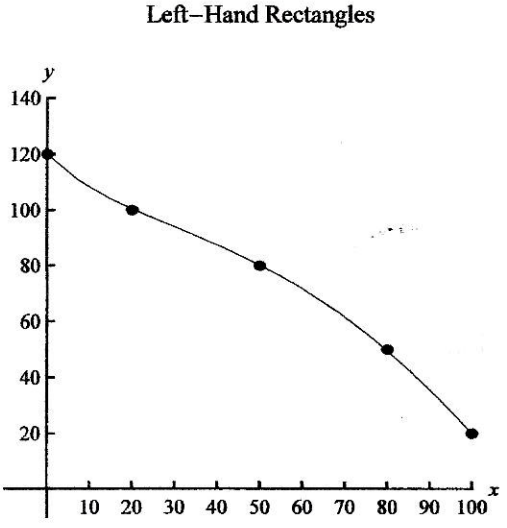
x	$f(x)$
0	20
30	30
40	40
50	55
70	65
90	70

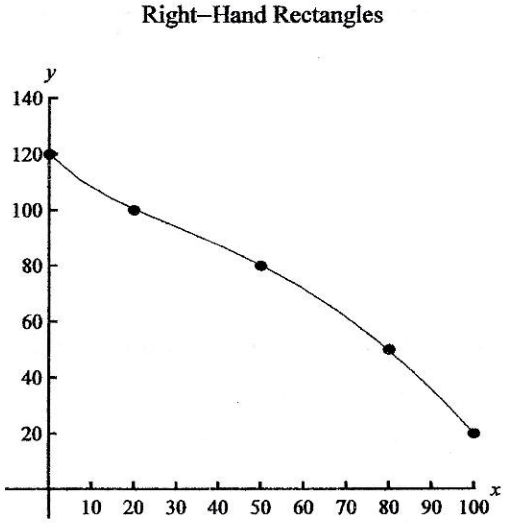




b.

x	$f(x)$
0	120
20	100
50	80
80	50
100	20

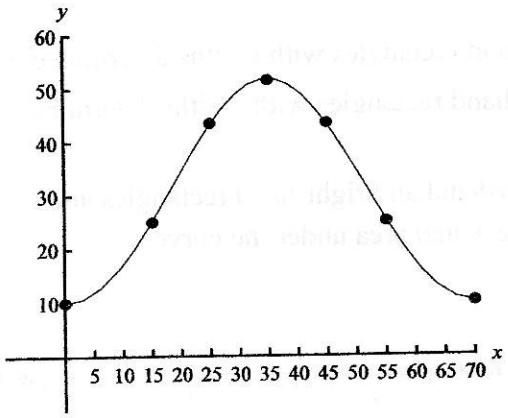




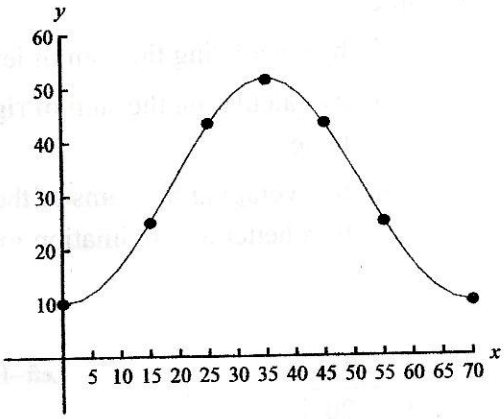
c.

x	$f(x)$
0	10
15	25
25	45
35	50
45	45
55	25
70	10

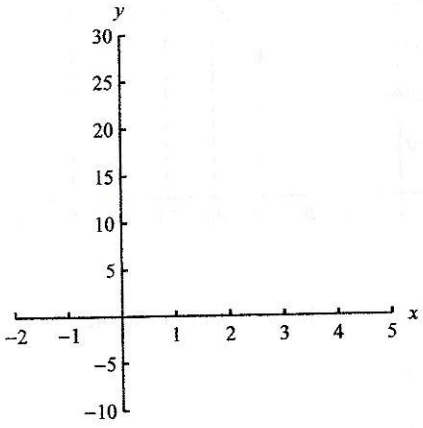
Left–Hand Rectangles



Right–Hand Rectangles



3. a. Graph the function $y = x^2 + 5$ on the interval $0 \le x \le 5$.



b. Complete the table of values for the function.

x	$f(x)$
0	
2	
4	
5	

c. Use the data in the table to approximate the area under the graph of the function by calculating the sum of left-hand rectangles with widths indicated by the data table.

d. Use the data in the table to approximate the area under the graph of the function by calculating the sum of the right-hand rectangles with widths indicated by the data table.

e. Calculate the average of your answers to parts (c) and (d).