# Applying Accumulation

1. An oil tanker collided with a Coast Guard cutter off the California coast. The disabled tanker was spilling oil from its damaged hull. The rate of flow of oil into the Pacific Ocean was measured at several different time intervals. The rate of flow is a non-increasing continuous function. The rates in hundreds of gallons per hour are listed in the table below.

9:00	10:00	11:00	12:00	1:00	2:00	3:00	4:00	5:00	6:00	7:00
4	4	3.8	3.6	3	2	0.6	0.3	0.2	0.2	0

a. Estimate the total amount of oil that spilled during the 10 hour period by using left-hand rectangles. Use correct units and explain your answer in terms of this situation.


Time in Hours
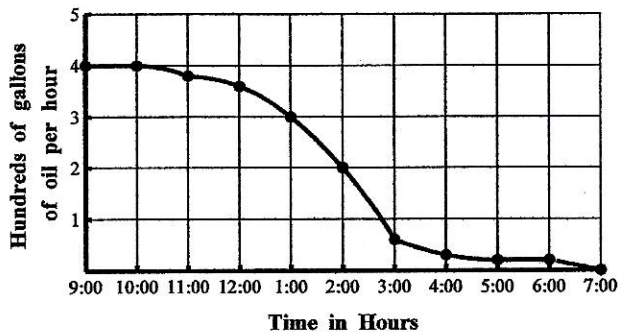
b. Estimate the total amount of oil that spilled during the 10 hour period covered in the table by using right-hand rectangles. Use correct units and explain your answer in terms of this situation.

Time in Hours
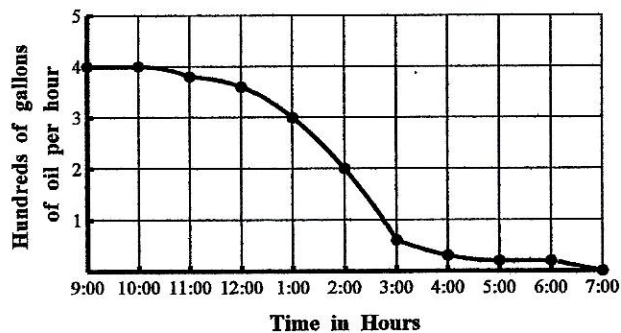
c. Environmentalists and the oil company are analyzing the situation. Who would use which method and why?

d. What could be done to improve the accuracy of these estimates?

e. An approximation of 10 trapezoids with equal widths can be determined by averaging the left- and right-hand approximations from parts (a) and (b). What is the approximation using 10 trapezoids?

f. Estimate the amount of oil spilled during the 10 hour period using 5 midpoint rectangles of equal length.
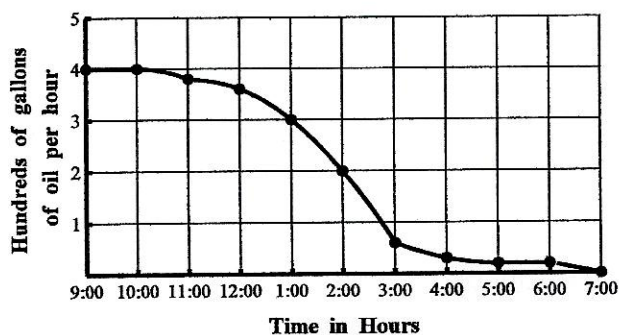
2. The amount of money it takes to dig a tunnel equals the length of the tunnel times the cost per unit length. If the cost per unit length is a constant, calculating the cost to dig a tunnel poses no problem; however, the price per unit length increases as the tunnel gets longer because of the expense of carrying in workers and tools and carrying out dirt and rock. Assume that the dollar price per foot varies according to the table. For example, for a tunnel 70 feet long, the cost is $500 per foot. For a tunnel 120 feet long, the cost is $500 per foot for the first 100 feet and $820 per foot for the remaining 20 feet.

   Note: The cost for up to and including 100 feet is $500/ft, etc.

Length	0-100	Over 100-200	Over 200-300	Over 300-400	Over 400-500	Over 500-600	Over 600-700	Over 700-800	Over 800-900	Over 900-1000
Cost per additional foot	500	820	1180	1580	2020	2500	3020	3580	4180	4820

a. What is the cost for the 70 foot tunnel?

b. What is the cost for the 120 foot tunnel?

3. A mining company wants to dig a silver mine with a length of 250 meters horizontally into the side of a mountain. The price per meter for digging the mine increases as the mine shaft gets longer because it is more expensive to bring out the dirt and the rock and to shore up the tunnel. Suppose the price, P, in dollars per meter for digging at a point x meters from the entrance is given by $P = 100 + 0.06x^2$.

    a. What is the parent function for P?

    b. What are the units for the area under the graph?

    c. Write a Riemann sum for five left-hand rectangles of equal width. Estimate the area under the graph with these five left-hand rectangles.

    d. Is your estimate an overestimate or an underestimate? Explain your answer.

    e. Write a Riemann sum for five right-hand rectangles of equal width. Estimate the area under the graph with these five right-hand rectangles.

    f. Is your estimate an overestimate or an underestimate? Explain your answer.

    g. Estimate the area under the graph with five trapezoids of equal height. Is your answer an overestimate or an underestimate?

    h. Write a Riemann sum for five midpoint rectangles of equal width. Estimate the area under the graph with these five midpoint rectangles.

    i. Is your estimate an overestimate or an underestimate? Explain your answer.

    j. Give an estimate for 500 midpoint rectangles. Show your calculator set up in mathematical notation (not calculator syntax) and your answer. Is your estimate an overestimate or an underestimate? Explain your answer

c. Sketch a graph using the data in the table. Identify the type of graph (i.e. curve, line, step function, etc.).
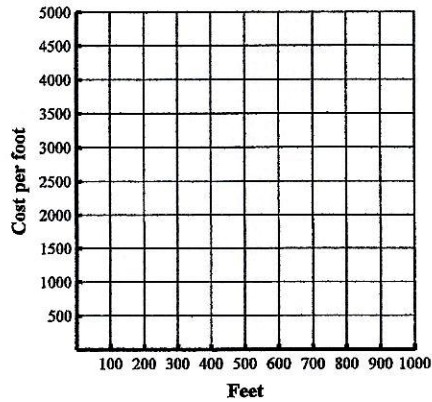
d. Describe the area under the graph in terms of the problem situation.

e. What is the exact total cost for digging a tunnel 1000 feet long?

f. How much money can be saved by starting the 1000 foot tunnel from both ends and making the two halves meet in the middle?