# Vectors in Geometry

1.  For each of the following problems draw the vectors on a coordinate plane and add the vectors graphically using the head-to-tail method. Calculate the magnitude of the resultant vector and give the direction angle for the resultant vector.

    a)  Vector *u* has a magnitude of 3 miles and is directed due east. Vector *v* has a magnitude of 3 miles and is directed due north. What are the magnitude and the direction angle for the resultant vector?

    b)  Vector *u* has a magnitude of 500 feet and is directed due south. Vector *v* has a magnitude of 500 feet and is directed due west. What are the magnitude and the direction angle for the resultant vector?

    c)  Vector *u* has a magnitude of $5\sqrt{3}$ feet and is directed due west. Vector *v* has a magnitude of 5 feet and is directed due north. What are the magnitude and the direction angle for the resultant vector?

    d)  Vector *u* has a magnitude of 300 miles and is directed due east. Vector *v* has a magnitude of $300\sqrt{3}$ miles and is directed due north. What are the magnitude and the direction angle for the resultant vector?

    e)  Vector *u* has a magnitude of 9 cm and is directed to the right. Vector *v* has a magnitude of $9\sqrt{3}$ cm and is directed downward. What are the magnitude and the direction angle for the resultant vector?

    f)  Vector *u* has a magnitude of 15 meters and is directed to the left. Vector *v* has a magnitude of 3 meters and is directed to the right. What are the magnitude and the direction angle for the resultant vector?

    g)  Vector *u* has a magnitude of 3 cm and is directed upward. Vector *v* has a magnitude of 11 cm and is directed downward. What are the magnitude and the direction angle for the resultant vector?

    h)  Vector *u* has a magnitude of 10 inches with a direction angle of 45°. Vector *v* has a magnitude of 10 inches and a direction angle of 225°. What are the magnitude and the direction angle for the resultant vector?

i)     Vector **u** has a magnitude of $10\sqrt{2}$ meters and is directed to the southwest. Vector **v** has a magnitude of 10 meters and is directed eastward. What are the magnitude and the direction angle for the resultant vector?

j)     Vector **u** has a magnitude of 24 miles with a direction angle of 150°. Vector **v** has a magnitude of $12\sqrt{3}$ miles with a direction angle of 0°. What are the magnitude and the direction for the resultant vector?

2.    Let **c** be the resultant of the two vectors **a** and **b**. Vector **b** has a magnitude of $b$ units directed to the east, and vector **a** has as a magnitude of $a$ units directed to the west. What are the three possibilities for the resultant's direction angle and magnitude?

3.    The vectors given below represent forces acting on a point at the origin of a coordinate plane. Determine the direction and magnitude of the resultant vector.

a)     Vector **a** has a magnitude of 10 lbs and a direction angle of 60°, and vector **b** has a magnitude of 10 lbs and a direction angle of 300°.

b)     Vector **a** has a magnitude of 24 kg and a direction angle of 0°, and vector **b** has a magnitude of 12 kg and a direction angle of 120°.

c)     Vector **a** has a magnitude of $10\sqrt{2}$ lbs and a direction angle of 45°, and vector **b** has a magnitude of $10\sqrt{2}$ lbs and a direction angle of 135°

4.    Given that $O$ is a point at the origin on which two forces are acting, what force must be combined with a force of 1 lb at a direction angle of 0° in order for the resultant force to have a magnitude of $\sqrt{3}$ lb. and a direction angle of 90°?

5.    Given that $O$ is a point at the origin on which two forces are acting, what force must be combined with a force of 20 kg at a direction angle of 60° in order for the resultant force to have a magnitude of $20\sqrt{3}$ kg and a direction angle of 90°?

6.    Determine two forces with direction angles of 90° and 180°, respectively, that result in a vector $b$ units in magnitude with a direction angle of 120°.

In mathematics the addition of two vectors is often indicated by completing a triangle instead of creating a parallelogram. The technique is referred to as a head-to-tail method. The first vector is drawn with its initial point at the origin and the second vector is drawn with its initial point at the terminal point of the first vector. (The initial point of a vector is called the tail of the vector and the terminal point is called the head.) To add the vectors, draw in the third side to complete the triangle. The third side, the resultant vector, is the sum of the two vectors. Its tail is the same as the first vector's tail. Its head is the same as the head of the second vector. The direction angle is the angle measured counter-clockwise from the positive x-axis to the resultant vector. See the drawings below.
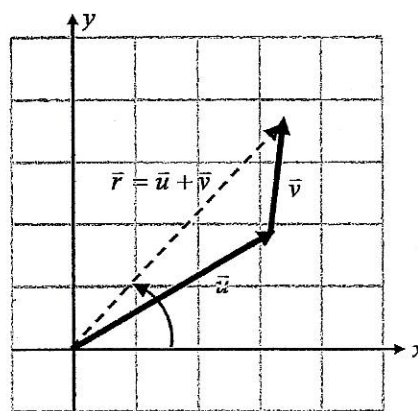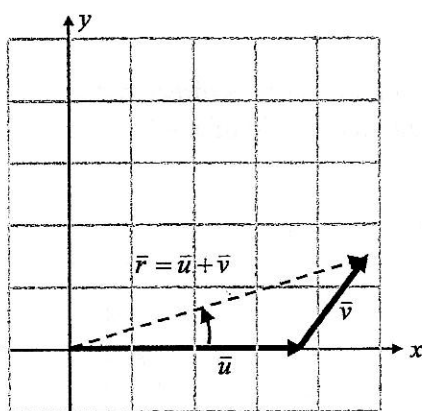


When vectors have the same or the opposite direction, the resultant vector will lie on the same line as the two given vectors. The head-to-tail method will not form a triangle; however, the resultant will still go from the first vector's tail to the second vector's head.

1. On a coordinate plane, sketch a vector with
   a) a magnitude of 4 and a direction angle of 90°.
   b) a magnitude of 3 and a direction of 0°
   c) a magnitude of 5 and a direction of 180°.
   d) a magnitude of $4\sqrt{2}$ and a direction of −45°.
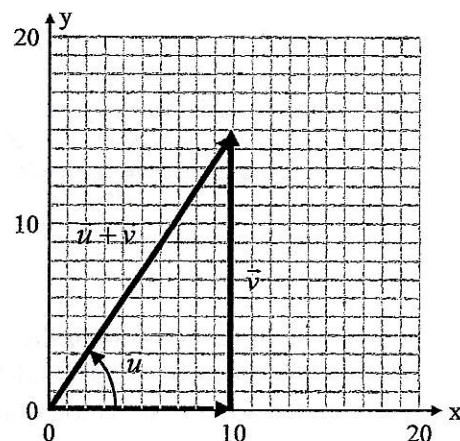   e) a magnitude of $5\sqrt{2}$ and a direction of 225°.
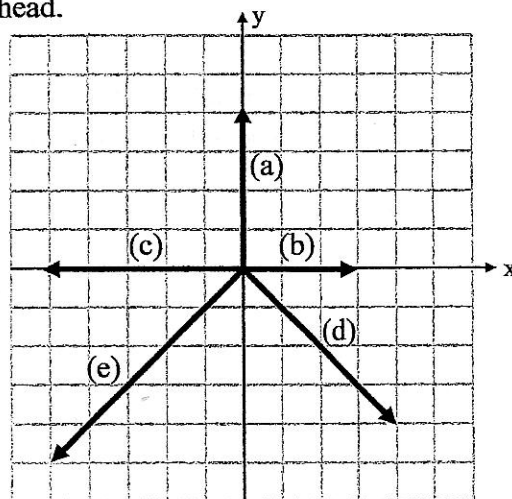
2.  On a coordinate plane, sketch a vector
    a) directed east with a magnitude of 5 feet.
    b) directed south with a magnitude of 4 feet.
    c) directed northwest with a magnitude of $3\sqrt{2}$ feet .
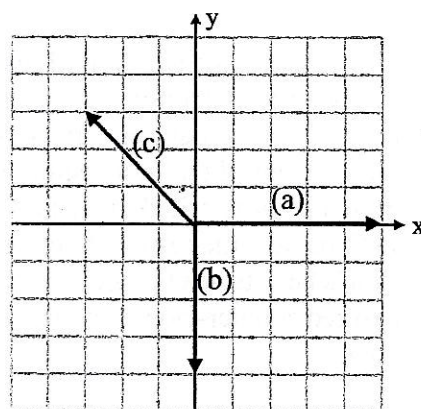


3.  Vector $\bar{u}$ is directed due east with a magnitude of 10 feet and $\bar{v}$ is directed due north with a magnitude of 15 feet   Draw $\bar{u}+\bar{v}$ and determine the magnitude of $\bar{u}+\bar{v}$.

    The magnitude is $\sqrt{10^2+15^2} = 5\sqrt{13}$ feet.



4.  What is the resultant of two forces if the first force is directed downward with a magnitude of 10 pounds, and the second is directed to the left with a magnitude of 10 pounds?

    The resultant is a vector with a magnitude of $10\sqrt{2}$ pounds and a direction angle of $225°$.