Henry is performing an experiment to introduce his sister, Liza, to the calculus topic of accumulation. He turns on the outside water to full force and then places a bucket under the faucet. When he is satisfied with the amount of water in the bucket, he removes the bucket from beneath the faucet. He removes the plug from the hole in the bottom of the bucket and uses a flow meter to measure how fast the water is draining out of the bucket. Since the bottom of the bucket is rusty, the hole continuously becomes larger as the water runs out, causing the rate at which the water runs out to continually increase. He measures the flow rate of the water out of the bucket in ounces per second at one second intervals and provides Liza with a graph and table of his data.
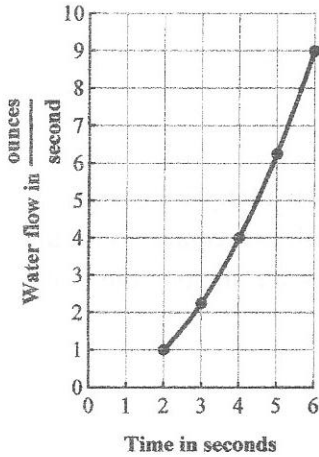
Time in seconds	Ounces of water per second
2	1
3	$2\frac{1}{4}$
4	4
5	$6\frac{1}{4}$
6	9

Henry challenges Liza to approximate the number of ounces of water that leak out of the bucket between 2 and 6 seconds using left-hand rectangles.
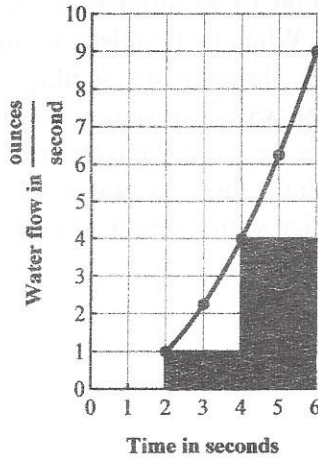
1. If one left-hand rectangle is used as shown on the graph, approximately how many ounces of water have leaked out of the bucket between 2 and 6 seconds? Show the work that leads to your answer and include the units as part of the process and as a part of the answer.
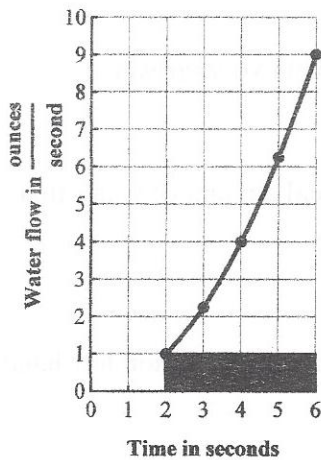

base (height) = Area of the rectangle

$$4 \text{ sec}\left(1\frac{\text{oz}}{\text{sec}}\right) = \underline{\hspace{2cm}}$$

2. If two left-hand rectangles of equal width are used as shown on the graph, approximately how many ounces of water have leaked out of the bucket during the four seconds? Show the work that leads to your answer and include the units as part of the process and as part of the answer.
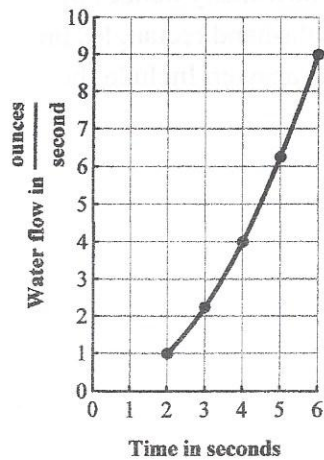

3. If four left-hand rectangles of equal width are used to approximate the answer, how many ounces of water have leaked out of the bucket during the four seconds? Sketch the four left-hand rectangles on the grid provided and then calculate the answer. Show the work that leads to your answer. Include the units as part of the process and as part of the answer.
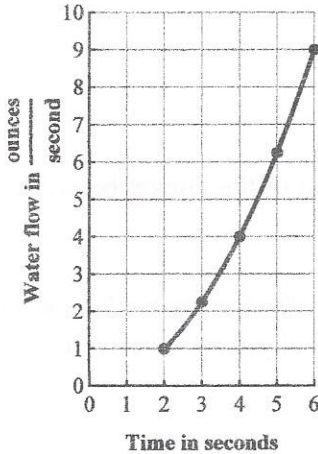

4. a. Which of the approximations using left-hand rectangles are too low? Explain your answer.

   b. Which of the three approximations provides the answer closest to the actual amount of water that leaks out of the bucket? Explain your answer.

   c. If the Henry could collect data every millisecond from 2 to 6 seconds and draw 4 million left-hand rectangles, would the approximation be too low? Explain your answer.

Henry asks Liza to repeat the process using right-hand rectangles instead of left-hand rectangles.


| Time in seconds	Ounces of water per second
2	1
3	$2\frac{1}{4}$
4	4
5	$6\frac{1}{4}$
6	9

5. If one right-hand rectangle is used as shown on the graph, approximately how many ounces of water have leaked out of the bucket between 2 and 6 seconds? Show the work that leads to your answer. Include the units as part of the process and as part of the answer.
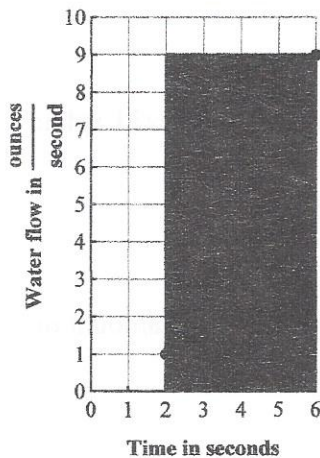
6. If two right-hand rectangles of equal width are used as shown on the graph, approximately how many ounces of water have leaked out of the bucket during the four seconds? Show the work that leads to your answer. Include the units as part of the process and as part of the answer.
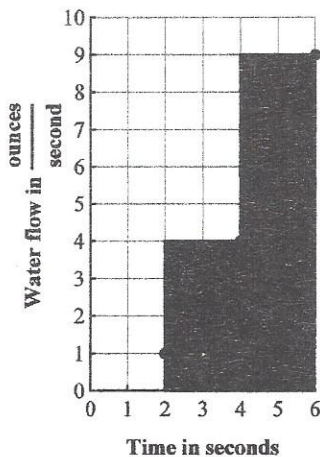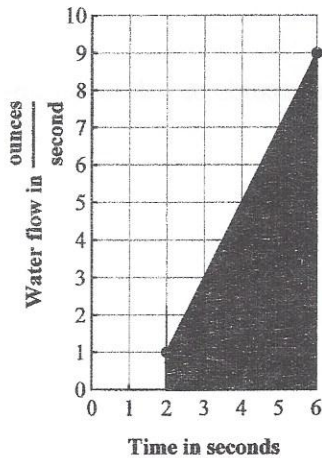
7. If four right-hand rectangles of equal width are used to approximate the answer, how many ounces of water have leaked out of the bucket between 2 and 6 seconds? Sketch the four right-hand rectangles on the grid provided and then calculate the answer. Show the work that leads to your answer. Include the units as part of the process and as part of the answer.
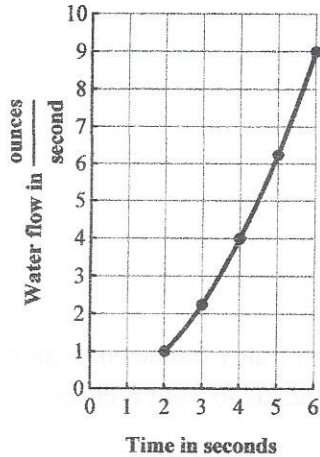

8. a. Are Liza's approximations using right-hand rectangles overestimates or underestimates? Explain your answer.

   b. Which of the three approximations using right-hand rectangles is closest to the actual amount of water that leaks out of the bucket? Explain your answer.

   c. If Henry could collect data every millisecond from 2 to 6 seconds and draw 4 million right-hand rectangles, would the approximation be too high or too low? Explain your answer.

Henry asks Liza to approximate the amount of water that leaks out of the bucket using trapezoids drawn between the curve and the *x*-axis and to compare the new method to the previous approximations.

9. If one trapezoid is used as shown on the following graph, approximately how many ounces of water have leaked out of the bucket between 2 and 6 seconds? Show the work that leads to your answer. Include the units as part of the process and as part of the answer.

10. Average the approximations for the number of ounces of water that have leaked out of the bucket during the four seconds using one left-hand rectangle (see question 1) and using one right-hand rectangle (see question 5), and then compare the answer to the approximation using one trapezoid.

11. If two trapezoids are used as shown on the graph, approximately how many ounces of water have leaked out of the bucket between 2 and 6 seconds? Show the work that leads to your answer. Include the units as part of the process and as a part of the answer.
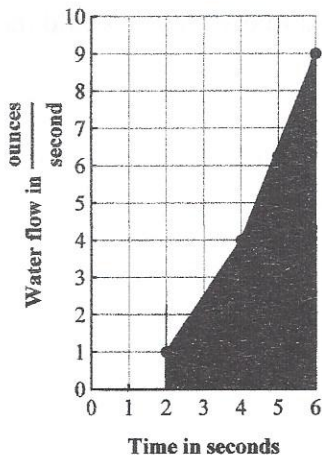
12. Average the approximations for the number of ounces of water that have leaked out of the bucket during the four second time period using two left-hand rectangles (see question 2) and using two right-hand rectangles (see question 6), and then compare the answer to the approximation using two trapezoids.

Liza tells Henry that she knows a shortcut for determining the area using four trapezoids based on her answers for using the four left-hand and four right-hand rectangles.

13. State Liza's shortcut in your own words.

14. Using the shortcut and four trapezoids of equal width, what is the approximation for the number of ounces of water that have leaked out of the bucket between 2 and 6 seconds?

15. a. Are Liza's estimates using trapezoids an overestimate or an underestimate? Explain your answer.

    b. If Henry could collect the data and could draw 1 million trapezoids from 2 to 6 seconds, would the estimate be too high or too low? Explain your answer.